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Delay-Dependent H_∞ Filtering for Markovian Jump Time-Delay Systems: A Piecewise Analysis Method

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Abstract A delay-dependent H_∞ filtering for Markovian jump systems with time-varying delays is studied based on a piecewise analysis approach. Firstly, by exploiting delay partitioning-based Lyapunov function, a new delay-dependent criterion is derived for the H_∞ performance analysis of the filtering-error systems, which can lead to much less conservative analysis results. Secondly, based on the criterion obtained, the gain of filter can be obtained in terms of linear matrix inequalities (LMIs). Finally, numerical examples are given to demonstrate the effectiveness of the proposed method.

Keywords Time delay systems · Piecewise analysis method · H_∞ filter · Markovian jump systems

1 Introduction

During the past few decades, Markovian jump systems (MJSs) have attracted much attention [5, 16, 18, 25, 27, 31, 37, 38]. Typically, MJSs can be regarded as a special

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class of hybrid systems with finite operation modes whose structures are subject to random abrupt changes. The system parameters usually jump among finite modes, and the mode switching is governed by a Markov process. MJSs have many applications, such as fault-tolerant flight control systems, economic systems, solar thermal receivers, and power systems. Over the past decades, a great number of important results related to such systems have been reported in the literature [17, 23, 24, 28, 29, 39, 42] and the references therein.

It is worth mentioning that the H_∞ filtering technique introduced in [4] has received increasing attention, see, for example, [6, 14, 35, 36]. The H_∞ filtering problem is to design an estimator to estimate the unknown state combination via output measurement, which guarantees that the L_2-induced gain from the external disturbance to the estimation error is less than a prescribed level. In recently years, the H_∞ filtering for time-delay systems has also received increasable attention since time delays are frequently encountered in many dynamic systems such as chemical or process control systems and networked control systems, and it is often a source of instability and oscillation in a filter system.

As of now, the stability criterion for the existence of a suitable filter can be classified into two categories, namely delay-independent filtering [19] and delay-dependent filtering [1, 11, 12, 15, 20, 32, 40, 41]. Since delay-independent criterion tends to be conservative, especially when the delay is small, much attention has been paid to the delay-dependent type. The main objective of the delay-dependent H_∞ filtering is to obtain a filter such that the filtering error system either allows a maximum delay bound for a fixed H_∞ performance or achieves a minimum H_∞ performance for a given delay bound.

This paper address the problem of H_∞ filter design for MJSs with interval time-varying delay. Based on a piecewise analysis method, the variation interval of the time delay is divided equally into two subintervals, by checking the variation of derivative of a Lyapunov functional in each subinterval, the convexity of matrix function method and the free-weighting matrix method are fully used in this paper. Different techniques are used in the derivation of the Lyapunov functional, and some novel delay-dependent criteria for asymptotic stability is derived in the form of LMIs. Compared with the existing method [9, 30], the conservativeness of the derived H_∞ performance analysis results is further reduced, and novel H_∞ filter design criteria are obtained. Examples used in [9, 30] are employed to show the effectiveness and less conservativeness of the proposed methods.

Notation: $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the n-dimensional Euclidean space and the set of $n \times m$ real matrices; the superscript "T" stands for matrix transposition; I is the identity matrix of appropriate dimension; $\|\cdot\|$ stands for the Euclidean vector norm or the induced matrix 2-norm as appropriate; the notation $X > 0$ (respectively, $X \geq 0$) for $X \in \mathbb{R}^{n\times n}$ means that the matrix X is real symmetric positive definite (respectively, positive semidefinite). When x is a stochastic variable, $\mathcal{E}\{x\}$ stands for the expectation of x. For a matrix B and two symmetric matrices A and C, $\left[\begin{smallmatrix} A & * \\ B & C \end{smallmatrix}\right]$ denotes a symmetric matrix, where $*$ denotes the entries implied by symmetry.

2 Systems Description and Preliminaries

Fix a probability space $(\Omega, \mathcal{F}, \mathcal{P})$ and consider the following class of linear systems with Markovian jump parameters and time-varying delays (Σ):

$$\begin{cases} \dot{x}(t) = A(\theta_t)x(t) + A_d(\theta_t)x(t-\tau(t)) + A_\omega(\theta_t)\omega(t), \\ y(t) = C(\theta_t)x(t) + C_d(\theta_t)x(t-\tau(t)) + C_\omega(\theta_t)\omega(t), \\ z(t) = L(\theta_t)x(t) + L_d(\theta_t)x(t-\tau(t)) + L_\omega(\theta_t)\omega(t), \\ x(t) = \phi(t), \quad \forall t \in [-\tau_M, -\tau_m], \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $y(t) \in \mathbb{R}^r$ is the measurement vector, $\omega(t) \in L_2[0, \infty)$ is the exogenous disturbance signal, $z(t) \in \mathbb{R}^p$ is the signal to be estimated, $\{\theta_t\}$ is a continuous-time Markovian process with right-continuous trajectories and taking values in a finite set $\mathcal{S} = \{1, 2, \dots, \mathcal{N}\}$ with stationary transition probabilities

$$\mathrm{Prob}\{\theta_{t+h} = j | \theta_t = i\} = \begin{cases} \pi_{ij}h + o(h), & i \neq j, \\ 1 + \pi_{ii}h + o(h), & i = j. \end{cases} \tag{2}$$

In the above, $h > 0$, $\lim_{h\to 0} \frac{o(h)}{h} = 0$, and $\pi_{ij} \geq 0$ for $j \neq i$ is the transition rate from mode i at time t to the mode j at time $t + h$, and

$$\pi_{ii} = -\sum_{j=1, j\neq i}^{N} \pi_{ij}. \tag{3}$$

In the system given by (1), the time delay $\tau(t)$ is a time-varying continuous function satisfying the following assumption:

$$0 \leq \tau_m \leq \tau(t) \leq \tau_M < \infty, \qquad \dot{\tau}(t) \leq \mu, \quad \forall t > 0, \tag{4}$$

where τ_m is the lower bound, and τ_M is the upper bound of the communication delay.

In this paper, we consider the following filter for system (1):

$$\begin{cases} \dot{\hat{x}}(t) = A(\theta_t)\hat{x}(t) + A_d(\theta_t)\hat{x}(t-\tau(t)) + G(\theta_t)(\hat{y}(t) - y(t)), \\ \hat{y}(t) = C(\theta_t)\hat{x}(t) + C_d(\theta_t)\hat{x}(t-\tau(t)), \\ \hat{z}(t) = L(\theta_t)\hat{x}(t) + L_d(\theta_t)\hat{x}(t-\tau(t)). \end{cases} \tag{5}$$

The set $\mathcal{S}$ comprises the various operation modes of system (1), and for each possible value of $\theta_t = i$, $i \in \mathcal{S}$, the matrices associated with "ith mode" will be denoted by

$$\begin{aligned} A_i &:= A(\theta_t = i), & A_{di} &:= A_d(\theta_t = i), & A_{\omega i} &:= A_\omega(\theta_t = i), \\ C_i &:= C(\theta_t = i), & C_{di} &:= C_d(\theta_t = i), & C_{\omega i} &:= C_\omega(\theta_t = i), \\ L_i &:= L(\theta_t = i), & L_{di} &:= L_d(\theta_t = i), & L_{\omega i} &:= L_\omega(\theta_t = i), \end{aligned}$$

where $A_i, A_{di}, A_{\omega i}, C_i, C_{di}, C_{\omega i}, L_i, L_{di}, L_{\omega i}$ are constant matrices for any $i \in \mathcal{S}$. It is assumed that the jumping process $\{\theta_t\}$ is accessible, i.e., the operation mode of system (Σ) is known for every $t \geq 0$.

Let $e(t)=\hat{x}(t)-x(t)$ and $\tilde{z}(t)=\hat{z}(t)-z(t)$. Then we have the following filtering error system:

$$\begin{cases} \dot{e}(t)=\bar{A}_i e(t)+\bar{A}_{di} e(t-\tau(t))+\bar{A}_{\omega i}\omega(t), \\ \tilde{z}(t)=L_i e(t)+L_{di} e(t-\tau(t))-L_{\omega i}\omega(t), \end{cases} \tag{6}$$

where $\bar{A}_i=A_i+G_iC_i$, $\bar{A}_{di}=A_{di}+G_iC_{di}$, $\bar{A}_{\omega i}=-A_{\omega i}-G_iC_{\omega i}$.

The H_∞ filtering problem addressed in this paper is to design a filter of form (5) such that

- The filtering error system (6) with $\omega(t)=0$ is exponentially stable;
- The H_∞ performance $\|\tilde{z}(t)\|_2<\gamma\|\omega(t)\|_2$ is guaranteed for all nonzero $\omega(t)\in L_2[0,\infty)$ and a prescribed $\gamma>0$ under the condition $e(t)=0\ \forall t\in[-\tau_M,-\tau_m]$.

The following lemmas and definitions are needed in the proof of our main results.

Lemma 1 [7] *For any constant matrix $R\in\mathbb{R}$, $R=R^T>0$, constant $\tau_M>0$, and vector function $\dot{x}:[-\tau_M,0]\to\mathbb{R}^n$ so that the following integration is well defined, the following condition holds*:

$$-\tau_M\int_{t-\tau_M}^{t}\dot{x}^T(s)R\dot{x}(s)\,ds\le\begin{bmatrix}x(t)\\ x(t-\tau_M)\end{bmatrix}^T\begin{bmatrix}-R & R\\ R & -R\end{bmatrix}\begin{bmatrix}x(t)\\ x(t-\tau_M)\end{bmatrix}. \tag{7}$$

Lemma 2 [34] *Suppose that $0\le\tau_m\le\tau(t)\le\tau_M$, and $\varXi_1$, $\varXi_2$, and $\varOmega$ are constant matrices of appropriate dimensions. Then*

$$\big(\tau(t)-\tau_m\big)\varXi_1+\big(\tau_M-\tau(t)\big)\varXi_2+\varOmega<0 \tag{8}$$

if and only if

$$(\tau_M-\tau_m)\varXi_1+\varOmega<0 \tag{9}$$

and

$$(\tau_M-\tau_m)\varXi_2+\varOmega<0. \tag{10}$$

Definition 1 System (6) is said to be exponentially stable in the mean-square sense (ESMSS), if there exist constants $\alpha>0$ and $\lambda>0$ such that for all $t>0$,

$$\mathcal{E}\left\{\|x(t)\|^2\right\}\le\alpha e^{-\lambda t}\sup_{-\tau_M<s<0}\left\{\|\phi(s)\|^2\right\}. \tag{11}$$

Definition 2 For a given function $V:C_{F_0}^b([-\tau_M,0],R^n)\times S\to R$, its infinitesimal operator $\mathcal{L}$ [13] is defined as

$$\mathcal{L}V(x_t)=\lim_{\Delta\to0^+}\frac{1}{\Delta}\big[\mathcal{E}\big(V(x_{t+\Delta}|x_t)-V(x_t)\big)\big]. \tag{12}$$

3 Main Results

In this section, we will concentrate our attention on the performance analysis for system (6) for $\tau(t)$ satisfying (4).

Similarly to [33], we divide the variation interval of the delay into l parts with equal length. Define

$$\tau_i = \tau_m + \frac{i(\tau_M - \tau_m)}{l}, \quad i = 1, 2, \dots, l. \tag{13}$$

Then, $[\tau_m, \tau_M] = [\tau_m, \tau_1] \bigcup_{i=1}^{l-1} (\tau_i, \tau_{i+1}]$. In the proof of our main results, we only discuss the case where $l = 2$. From the following discussion it can be seen that the proposed method of this paper can also be easily extended to the cases with l being any finite integer.

Define

$$\delta = \frac{\tau_M - \tau_m}{2}.$$

Then

$$\tau_1 = \tau_m + \delta = \frac{\tau_m + \tau_M}{2}.$$

Furthermore, define a new vector

$$\zeta^T(t) = \begin{bmatrix} e^T(t) & e^T\big(t - \tau(t)\big) & e^T(t - \tau_m) & e^T(t - \tau_1) & e^T(t - \tau_M) & \omega^T(t) \end{bmatrix}$$

and two matrices

$$\Gamma_1 = \begin{bmatrix} \bar{A}_i & \bar{A}_{di} & 0 & 0 & 0 & \bar{A}_{\omega i} \end{bmatrix}, \qquad \Gamma_2 = \begin{bmatrix} L_i & -L_{di} & 0 & 0 & 0 & -L_{\omega i} \end{bmatrix}.$$

Rewrite (6) as

$$\begin{cases} \dot{e}(t) = \Gamma_1 \zeta(t), \\ \tilde{z}(t) = \Gamma_2 \zeta(t). \end{cases} \tag{14}$$

On the basis of (14), we get the following results.

Theorem 1 *For some given constants $0 \le \tau_m \le \tau_M$ and γ, system (6) is ESMSS with a prescribed H_∞ performance γ if there exist $P_i > 0$, $Q_1 > 0$, $Q_2 > 0$, $Q_3 > 0$, $Q_{4i} > 0$, $R_1 > 0$, $R_2 > 0$, $R_3 > 0$, $Z_1 > 0$, $Z_2 > 0$, $Z_3 > 0$, M_{ik}, N_{ik}, T_{ik}, and S_{ik} ($i \in \mathcal{S}, k = 1, 2, \dots, 6$) with appropriate dimensions such that the following matrix inequalities hold*:

$$\Psi = \begin{bmatrix} \Psi_{11} + \Gamma + \Gamma^T & * & * \\ \hat{\Psi}_{21} & \Psi_{22} & * \\ \Psi_{31}(s) & 0 & -R_2 \end{bmatrix} < 0, \quad s = 1, 2, \tag{15}$$

$$\Omega = \begin{bmatrix} \Omega_{11} + \Upsilon + \Upsilon^T & * & * \\ \Psi_{21} & \Psi_{22} & * \\ \Omega_{31}(s) & 0 & -R_3 \end{bmatrix} < 0, \quad s = 1, 2, \tag{16}$$

$$\sum_{j=1}^{N} \pi_{ij} Q_{4j} \leq Z_k, \quad k = 1, 2, 3, \tag{17}$$

where

$$\Psi_{11} = \begin{bmatrix} \Pi_1 & * & * & * & * & * \\ \bar{A}_{di}^T P_i & -(1-\mu) Q_{4i} & * & * & * & * \\ R_1 & 0 & -Q_1 - R_1 & * & * & * \\ 0 & 0 & 0 & -Q_2 - \frac{R3}{\delta} & * & * \\ 0 & 0 & 0 & \frac{R3}{\delta} & -Q_3 - \frac{R3}{\delta} & * \\ \bar{A}_{\omega i}^T P_i & 0 & 0 & 0 & 0 & -\gamma^2 I \end{bmatrix},$$

$$\Omega_{11} = \begin{bmatrix} \Pi_1 & * & * & * & * & * \\ \bar{A}_{di}^T P_i & -(1-\mu) Q_{4i} & * & * & * & * \\ R_1 & 0 & -Q_1 - R_1 - \frac{R2}{\delta} & * & * & * \\ 0 & 0 & \frac{R2}{\delta} & -Q_2 - \frac{R2}{\delta} & * & * \\ 0 & 0 & 0 & 0 & -Q_3 & * \\ \bar{A}_{\omega i}^T P_i & 0 & 0 & 0 & 0 & -\gamma^2 I \end{bmatrix},$$

$$\Psi_{21} = \begin{bmatrix} \tau_m R_1 \bar{A}_i & \tau_m R_1 \bar{A}_{di} & 0 & 0 & 0 & \tau_m R_1 \bar{A}_{\omega i} \\ \sqrt{\delta} R_2 \bar{A}_i & \sqrt{\delta} R_2 \bar{A}_{di} & 0 & 0 & 0 & \sqrt{\delta} R_2 \bar{A}_{\omega i} \\ \sqrt{\delta} R_3 \bar{A}_i & \sqrt{\delta} R_3 \bar{A}_{di} & 0 & 0 & 0 & \sqrt{\delta} R_3 \bar{A}_{\omega i} \\ L_i & L_{di} & 0 & 0 & 0 & -L_{\omega i} \end{bmatrix},$$

$$\Psi_{22} = \operatorname{diag}\{-R_1, -R_2, -R_3, -I\},$$

$$\Psi_{31}(1) = \sqrt{\delta} M_i^T, \qquad \Psi_{31}(2) = \sqrt{\delta} N_i^T,$$

$$\Omega_{31}(1) = \sqrt{\delta} T_i^T, \qquad \Omega_{31}(2) = \sqrt{\delta} S_i^T,$$

$$\Gamma = \begin{bmatrix} 0 & -M_i + N_i & M_i & -N_i & 0 & 0 \end{bmatrix},$$

$$\Upsilon = \begin{bmatrix} 0 & -T_i + S_i & 0 & T_i & -S_i & 0 \end{bmatrix},$$

$$\Pi_1 = P_i \bar{A}_i + \bar{A}_i^T P_i + Q_1 + Q_2 + Q_3 + Q_{4i} - R_1 + \tau_m Z_1 + \delta Z_2 + \delta Z_3 + \sum_{j=1}^{N} \pi_{ij} P_j.$$

Proof Let $x_t(s) = x(t+s)$, $-\tau(t) \leq s \leq 0$. Then, similarly to [2], $\{(x_t, \theta_t), t \geq 0\}$ is a Markov process. Construct a Lyapunov functional candidate as

$$V(x_t, \theta_t) = \sum_{i=1}^{4} V_i(x_t, \theta_t), \tag{18}$$

where

$$
\begin{aligned}
V_1(x_t,\theta_t) &= e^T(t)P(\theta_t)e(t),\\
V_2(x_t,\theta_t) &= \int_{t-\tau_m}^{t} e^T(s)Q_1e(s)\,ds + \int_{t-\tau_1}^{t} e^T(s)Q_2e(s)\,ds + \int_{t-\tau_M}^{t} e^T(s)Q_3e(s)\,ds\\
&\quad + \int_{t-\tau(t)}^{t} e^T(s)Q_4(\theta_t)e(s)\,ds,\\
V_3(x_t,\theta_t) &= \tau_m \int_{t-\tau_m}^{t}\int_{s}^{t} \dot e^T(v)R_1\dot e(v)\,dv\,ds + \int_{t-\tau_1}^{t-\tau_m}\int_{s}^{t} \dot e^T(v)R_2\dot e(v)\,dv\,ds\\
&\quad + \int_{t-\tau_M}^{t-\tau_1}\int_{s}^{t} \dot e^T(v)R_3\dot e(v)\,dv\,ds,\\
V_4(x_t,\theta_t) &= \int_{t-\tau_m}^{t}\int_{s}^{t} e^T(v)Z_1e(v)\,dv\,ds + \int_{t-\tau_1}^{t-\tau_m}\int_{s}^{t} e^T(v)Z_2e(v)\,dv\,ds\\
&\quad + \int_{t-\tau_M}^{t-\tau_1}\int_{s}^{t} e^T(v)Z_3e(v)\,dv\,ds.
\end{aligned}
$$

Let $\mathcal{L}$ be the weak infinite generator of the random process $\{x_t,\theta_t\}$. Then, for each $\theta_t = i$ $(i \in \mathcal{S})$, we have

$$
\begin{aligned}
\mathcal{L}\big[V(x_t,\theta_t)\big] \le e^T(t)\Bigg(&2P_i\bar A_i + Q_1 + Q_2 + Q_3 + Q_{4i}\\
&+ \tau_m Z_1 + \delta Z_2 + \delta Z_3 + \sum_{j=1}^{N}\pi_{ij}P_j\Bigg)e(t)\\
&+ 2e^T(t)P_i\bar A_{di}e\big(t-\tau(t)\big) + 2e^T(t)P_i\bar A_{\omega i}\omega(t)\\
&- e^T(t-\tau_m)Q_1e(t-\tau_m) - e^T(t-\tau_1)Q_2e(t-\tau_1)\\
&- e^T(t-\tau_M)Q_3e(t-\tau_M) - (1-\mu)e^T\big(t-\tau(t)\big)Q_{4i}e\big(t-\tau(t)\big)\\
&+ \int_{t-\tau(t)}^{t} e^T(s)\Bigg(\sum_{j=1}^{N}\pi_{ij}Q_{4j}\Bigg)e(s)\,ds\\
&+ \dot e^T(t)\big(\tau_m^2R_1 + \delta R_2 + \delta R_3\big)\dot e(t) - \tau_m\int_{t-\tau_m}^{t} \dot e^T(s)R_1\dot e(s)\,ds\\
&- \int_{t-\tau_1}^{t-\tau_m} \dot e^T(s)R_2\dot e(s)\,ds - \int_{t-\tau_M}^{t-\tau_1} \dot e^T(s)R_3\dot e(s)\,ds\\
&- \int_{t-\tau_m}^{t} e^T(s)Z_1e(s)\,ds - \int_{t-\tau_1}^{t-\tau_m} e^T(s)Z_2e(s)\,ds\\
&- \int_{t-\tau_M}^{t-\tau_1} e^T(s)Z_3e(s)\,ds.
\end{aligned}
\tag{19}
$$

Since

$$\int_{t-\tau(t)}^{t} e^T(s)\left(\sum_{j=1}^{N}\pi_{ij}Q_{4j}\right)e(s)\,ds = \int_{t-\tau(t)}^{t-\tau_1} e^T(s)\left(\sum_{j=1}^{N}\pi_{ij}Q_{4j}\right)e(s)\,ds$$
$$+\int_{t-\tau_1}^{t-\tau_m} e^T(s)\left(\sum_{j=1}^{N}\pi_{ij}Q_{4j}\right)e(s)\,ds$$
$$+\int_{t-\tau_m}^{t} e^T(s)\left(\sum_{j=1}^{N}\pi_{ij}Q_{4j}\right)e(s)\,ds \qquad (20)$$

from (17) and (20) we have

$$\int_{t-\tau(t)}^{t} e^T(s)\left(\sum_{j=1}^{N}\pi_{ij}Q_{4j}\right)e(s)\,ds - \int_{t-\tau_m}^{t} e^T(s)Z_1e(s)\,ds$$
$$-\int_{t-\tau_1}^{t-\tau_m} e^T(s)Z_2e(s)\,ds - \int_{t-\tau_M}^{t-\tau_1} e^T(s)Z_3e(s)\,ds$$
$$=\int_{t-\tau_m}^{t} e^T(s)\left(\sum_{j=1}^{N}\pi_{ij}Q_{4j}-Z_1\right)e(s)\,ds$$
$$+\int_{t-\tau_1}^{t-\tau_m} e^T(s)\left(\sum_{j=1}^{N}\pi_{ij}Q_{4j}-Z_2\right)e(s)\,ds$$
$$+\int_{t-\tau(t)}^{t-\tau_1} e^T(s)\left(\sum_{j=1}^{N}\pi_{ij}Q_{4j}\right)e(s)\,ds - \int_{t-\tau_M}^{t-\tau_1} e^T(s)Z_3e(s)\,ds$$
$$\le \int_{t-\tau(t)}^{t-\tau_1} e^T(s)Z_3e(s)\,ds - \int_{t-\tau_M}^{t-\tau_1} e^T(s)Z_3e(s)\,ds \le 0. \qquad (21)$$

Applying Lemma 1, we have

$$-\tau_m\int_{t-\tau_m}^{t}\dot{e}^T(s)R_1\dot{e}(s)\,ds \le \begin{bmatrix} e(t) \\ e(t-\tau_m)\end{bmatrix}^T \begin{bmatrix} -R_1 & R_1 \\ R_1 & -R_1\end{bmatrix}\begin{bmatrix} e(t) \\ e(t-\tau_m)\end{bmatrix}. \qquad (22)$$

Combining (19, 21, 22), we obtain

$$\mathcal{L}\big[V(x_t,\theta_t)\big] - \gamma^2\omega^T(t)\omega(t) + \tilde{z}^T(t)\tilde{z}(t)$$
$$\le e^T(t)\left(2P_i\bar{A}_i + Q_1 + Q_2 + Q_3 + Q_{4i} + \tau_m Z_1 + \delta Z_2 + \delta Z_3 + \sum_{j=1}^{N}\pi_{ij}P_j\right)e(t)$$
$$+2e^T(t)P_i\bar{A}_{di}e\big(t-\tau(t)\big) + 2e^T(t)P_i\bar{A}_{\omega i}\omega(t) - e^T(t-\tau_m)Q_1e(t-\tau_m)$$

$$
\begin{aligned}
&- e^T(t-\tau_1)Q_2e(t-\tau_1) - e^T(t-\tau_M)Q_3e(t-\tau_M) \\
&- (1-\mu)e^T\big(t-\tau(t)\big)Q_{4i}e\big(t-\tau(t)\big) + \dot{e}^T(t)\big(\tau_m^2R_1+\delta R_2+\delta R_3\big)\dot{e}(t) \\
&+ \begin{bmatrix} e(t) \\ e(t-\tau_m) \end{bmatrix}^T \begin{bmatrix} -R_1 & R_1 \\ R_1 & -R_1 \end{bmatrix} \begin{bmatrix} e(t) \\ e(t-\tau_m) \end{bmatrix} - \int_{t-\tau_1}^{t-\tau_m} \dot{e}^T(s)R_2\dot{e}(s)\,ds \\
&- \int_{t-\tau_M}^{t-\tau_1} \dot{e}^T(s)R_3\dot{e}(s)\,ds - \gamma^2\omega^T(t)\omega(t) + \zeta^T(t)\Gamma_2^T\Gamma_2\zeta(t). \qquad (23)
\end{aligned}
$$

It is noted that, for any $t \in R_+$, $\tau(t) \in [\tau_m, \tau_1]$ or $\tau(t) \in (\tau_1, \tau_M]$. Define two sets

$$\Omega_1 = \big\{t : \tau(t) \in [\tau_m, \tau_1]\big\}, \qquad (24)$$

$$\Omega_2 = \big\{t : \tau(t) \in (\tau_1, \tau_M]\big\}. \qquad (25)$$

In the following, we will discuss the variation of $\mathcal{L}V(x_t, \theta_t)$ in two cases, that is, $t \in \Omega_1$ or $t \in \Omega_2$.

Case 1: $t \in \Omega_1$, i.e., $\tau(t) \in [\tau_m, \tau_1]$.

By using Lemma 1 we have

$$-\int_{t-\tau_M}^{t-\tau_1} \dot{e}^T(s)R_3\dot{e}(s)\,ds \le \frac{1}{\delta}\begin{bmatrix} e(t-\tau_1) \\ e(t-\tau_M) \end{bmatrix}^T \begin{bmatrix} -R_3 & R_3 \\ R_3 & -R_3 \end{bmatrix} \begin{bmatrix} e(t-\tau_1) \\ e(t-\tau_M) \end{bmatrix}. \qquad (26)$$

Employing the free matrix method, we have

$$2\zeta^T(t)M_i\left[e(t-\tau_m) - e\big(t-\tau(t)\big) - \int_{t-\tau(t)}^{t-\tau_m} \dot{e}(s)\,ds\right] = 0, \qquad (27)$$

$$2\zeta^T(t)N_i\left[e\big(t-\tau(t)\big) - e(t-\tau_1) - \int_{t-\tau_1}^{t-\tau(t)} \dot{e}(s)\,ds\right] = 0, \qquad (28)$$

where

$$
\begin{aligned}
M_i^T &= \begin{bmatrix} M_{i1}^T & M_{i2}^T & M_{i3}^T & M_{i4}^T & M_{i5}^T & M_{i6}^T \end{bmatrix}, \\
N_i^T &= \begin{bmatrix} N_{i1}^T & N_{i2}^T & N_{i3}^T & N_{i4}^T & N_{i5}^T & N_{i6}^T \end{bmatrix}
\end{aligned}
$$

and $i \in \mathcal{S}$.

There exists R_2 such that

$$
\begin{aligned}
-2\zeta^T(t)M_i \int_{t-\tau(t)}^{t-\tau_m} \dot{e}(s)\,ds \le{}& \big(\tau(t)-\tau_m\big)\zeta^T(t)M_iR_2^{-1}M_i^T\zeta(t) \\
&+ \int_{t-\tau(t)}^{t-\tau_m} \dot{e}^T(s)R_2\dot{e}(s)\,ds, \qquad (29)
\end{aligned}
$$

$$-2\zeta^T(t)N_i\int_{t-\tau_1}^{t-\tau(t)}\dot{e}(s)\,ds \le \big(\tau_1-\tau(t)\big)\zeta^T(t)N_iR_2^{-1}N_i^T\zeta(t) + \int_{t-\tau_1}^{t-\tau(t)}\dot{e}^T(s)R_2\dot{e}(s)\,ds. \tag{30}$$

Adding (27) and (28) to the right of (23) and substituting (26), (29), and (30) into (23), we have

$$\begin{aligned}
&\mathcal{L}\big[V(x_t,\theta_t)\big]-\gamma^2\omega^T(t)\omega(t)+\tilde{z}^T(t)\tilde{z}(t)\\
&\quad\le \zeta^T(t)\begin{bmatrix}\Psi_{11}+\Gamma+\Gamma^T & *\\ \Psi_{21} & \Psi_{22}\end{bmatrix}\zeta(t)\\
&\qquad+\big(\tau(t)-\tau_m\big)\zeta^T(t)M_iR_2^{-1}M_i^T+\big(\tau_1-\tau(t)\big)\zeta^T(t)N_iR_2^{-1}N_i^T\zeta(t).
\end{aligned} \tag{31}$$

Using Lemma 2 and Schur complement, it is easy to see that (15) with $s=1,2$ are sufficient conditions to guarantee

$$\mathcal{L}\big[V(x_t,\theta_t)\big]-\gamma^2\omega^T(t)\omega(t)+\tilde{z}^T(t)\tilde{z}(t)<0. \tag{32}$$

Case 2: $t\in\Omega_2$, i.e., $\tau(t)\in(\tau_1,\tau_M]$.

By using Lemma 1 we have

$$-\int_{t-\tau_1}^{t-\tau_m}\dot{e}^T(s)R_2\dot{e}(s)\,ds \le \frac{1}{\delta}\begin{bmatrix}e(t-\tau_m)\\ e(t-\tau_1)\end{bmatrix}^T\begin{bmatrix}-R_2 & R_2\\ R_2 & -R_2\end{bmatrix}\begin{bmatrix}e(t-\tau_m)\\ e(t-\tau_1)\end{bmatrix}. \tag{33}$$

Employing the free matrix method, we have

$$2\zeta^T(t)T_i\left[e(t-\tau_1)-e\big(t-\tau(t)\big)-\int_{t-\tau(t)}^{t-\tau_1}\dot{e}(s)\,ds\right]=0, \tag{34}$$

$$2\zeta^T(t)S_i\left[e\big(t-\tau(t)\big)-e(t-\tau_M)-\int_{t-\tau_M}^{t-\tau(t)}\dot{e}(s)\,ds\right]=0, \tag{35}$$

where

$$\begin{aligned}
T_i^T&=\begin{bmatrix}T_{i1}^T & T_{i2}^T & T_{i3}^T & T_{i4}^T & T_{i5}^T & T_{i6}^T\end{bmatrix},\\
S_i^T&=\begin{bmatrix}S_{i1}^T & S_{i2}^T & S_{i3}^T & S_{i4}^T & S_{i5}^T & S_{i6}^T\end{bmatrix}
\end{aligned}$$

and $i\in\mathcal{S}$.

There exists R_3 such that

$$-2\zeta^T(t)T_i\int_{t-\tau(t)}^{t-\tau_1}\dot{e}(s)\,ds \le \big(\tau(t)-\tau_1\big)\zeta^T(t)T_iR_3^{-1}T_i^T\zeta(t) + \int_{t-\tau(t)}^{t-\tau_1}\dot{e}^T(s)R_3\dot{e}(s)\,ds, \tag{36}$$

$$-2\zeta^T(t)S_i\int_{t-\tau_M}^{t-\tau(t)}\dot{e}(s)\,ds \le \big(\tau_M-\tau(t)\big)\zeta^T(t)S_iR_3^{-1}S_i^T\zeta(t) + \int_{t-\tau_M}^{t-\tau(t)}\dot{e}^T(s)R_3\dot{e}(s)\,ds. \tag{37}$$

Adding (34) and (35) to the right of (23) and substituting (33), (36), and (37) into (23), we have

$$\begin{aligned}
&\mathcal{L}\big[V(x_t,\theta_t)\big]-\gamma^2\omega^T(t)\omega(t)+\tilde{z}^T(t)\tilde{z}(t)\\
&\quad\le \zeta^T(t)\begin{bmatrix}\Omega_{11}+\Upsilon+\Upsilon^T & *\\ \Psi_{21} & \Psi_{22}\end{bmatrix}\zeta(t)\\
&\qquad+\big(\tau(t)-\tau_1\big)\zeta^T(t)T_iR_3^{-1}T_i^T\zeta(t)+\big(\tau_M-\tau(t)\big)\zeta^T(t)S_iR_3^{-1}S_i^T\zeta(t).
\end{aligned} \tag{38}$$

Using Lemma 2 and Schur complement, it is easy to see that (16) with $s=1,2$ are sufficient conditions to guarantee

$$\mathcal{L}\big[V(x_t,\theta_t)\big]-\gamma^2\omega^T(t)\omega(t)+\tilde{z}^T(t)\tilde{z}(t)<0. \tag{39}$$

Then, the following inequality can be concluded:

$$\mathcal{E}\big\{\mathcal{L}V(x_t,i,t)\big\}<-\lambda_{\min}(\Psi,\Omega)\mathcal{E}\big\{\zeta^T(t)\zeta(t)\big\}. \tag{40}$$

Define a new function as

$$W(x_t,i,t)=e^{\epsilon t}V(x_t,i,t). \tag{41}$$

Its infinitesimal operator $\mathcal{L}$ is given by

$$\mathcal{W}(x_t,i,t)=\epsilon e^{\epsilon t}V(x_t,i,t)+e^{\epsilon t}\mathcal{L}V(x_t,i,t). \tag{42}$$

By the generalized Itô formula[13], we can obtain from (42) that

$$\mathcal{E}\big\{W(x_t,i,t)\big\}-\mathcal{E}\big\{W(x_0,i)\big\}=\int_0^t \epsilon e^{\epsilon s}\mathcal{E}\big\{V(x_s,i)\big\}\,ds + \int_0^t e^{\epsilon s}\mathcal{E}\big\{\mathcal{L}V(x_s,i)\big\}\,ds. \tag{43}$$

Then, using a method similar to that in [31], we can see that there exists a positive number α such that for $t>0$,

$$\mathcal{E}\big\{V(x_t,i,t)\big\}\le\alpha\sup_{-\tau_M\le s\le 0}\big\{\|\phi(s)\|^2\big\}e^{-\epsilon t}. \tag{44}$$

Since $V(x_t,i,t)\ge\{\lambda_{\min}(P_i)\}x^T(t)x(t)$, it can be shown from (44) that for $t\ge 0$,

$$\mathcal{E}\big\{x^T(t)x(t)\big\}\le\bar{\alpha}^{-\epsilon t}\sup_{-\tau_M\le s\le 0}\big\{\|\phi(s)\|^2\big\}, \tag{45}$$

where $\bar{\alpha}=\alpha/(\lambda_{\min}P_i)$. Recalling Definition 1, the proof can be completed. □

Remark 1 In the above proof, it should be noted that $V_4(x_t, \theta_t)$ is employed in the Lyapunov function and $\int_{t-\tau(t)}^{t} x^T(s)(\sum_{j=1}^{N} \pi_{ij} Q_{2j})x(s)\,ds$ is separated into three parts. From Sect. 5 containing the examples we can see that this method is less conservative than the existing ones [29, 30].

Remark 2 Theorem 1 provides a delay-dependent stability condition for MJS with interval time-varying delays.Throughout the proof of Theorem 1, it can be seen that the convexity property of the matrix inequality is treated in terms of Lemma 2, which need not enlarge $\tau(t)$ to τ_M; therefore the commonly existing conservatism caused by this kind of enlargement in [3, 10, 21, 22, 26] can be avoided, which will reduce the conservativeness of the result.

Remark 3 To further reduce the conservatism, we can divide the variation of the delay into k ($k \geq 3$) parts with equal length. It can be easily extended by the proposed method in Theorem 1. For the brevity of the analysis, we omit it here.

As a special case, we consider $\dot{\tau}(t) = 0$, that is, the time delay $\tau(t)$ is a constant. In this case, system (6) reduces to the system

$$\begin{cases} \dot{e}(t) = \bar{A}_i e(t) + \bar{A}_{di} e(t-\tau) + \bar{A}_{\omega i}\omega(t), \\ \tilde{z}(t) = L_i e(t) + L_{di} e(t-\tau) - L_{\omega i}\omega(t), \end{cases} \tag{46}$$

where τ denotes the constant time delay of the state in the system.

Using a method similar to that used in [5], the following result can be obtained.

Theorem 2 *For some given constants τ, d, and γ, system (46) is ESMSS with a prescribed H_∞ performance γ if there exist $P_i > 0$, $Q_1 > 0$, $Q_2 > 0$, $R_i > 0$, and $Z > 0$ ($i \in \mathcal{S}$) with appropriate dimensions such that the following matrix inequalities hold*:

$$\Psi = \begin{bmatrix} \Psi_{11} + R_i + \frac{\tau}{d} Z & * & * \\ \Psi_{21} & \Psi_{22} & * \\ \Psi_{31} & \Psi_{32} & \Psi_{33} \end{bmatrix} < 0, \tag{47}$$

$$\sum_{j=1}^{N} \pi_{ij} R_j \leq Z, \tag{48}$$

where

$$\Psi_{11} = \begin{bmatrix} \Gamma & * & * & * & * \\ d * Q_1 & -R_{i11} - d * Q_1 & * & * & * \\ 0 & -R_{i21} & -R_{i22} & * & * \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & -R_{i(d-1)1} & -R_{i(d-1)2} & \cdots & -R_{i(d-1)(d-1)} \end{bmatrix},$$

$$\Psi_{21} = \begin{bmatrix} \bar{A}_{di}^T P_i + Q_2 & -R_{id1} & -R_{id2} & \cdots & -R_{id(d-1)} \\ \bar{A}_{\omega i}^T P_i & 0 & 0 & \cdots & 0 \end{bmatrix},$$

$$\Psi_{22} = \mathrm{diag}\{-R_{idd} - Q_2, -\gamma^2 I\},$$

$$\Psi_{31} = \begin{bmatrix} \frac{\tau}{\sqrt{d}} Q_1 \bar{A}_i & 0 & 0 & \cdots & 0 \\ \tau Q_2 \bar{A}_i & 0 & 0 & \cdots & 0 \\ L_i & 0 & 0 & \cdots & 0 \end{bmatrix},$$

$$\Psi_{32} = \begin{bmatrix} \frac{\tau}{\sqrt{d}} Q_1 \bar{A}_{di} & \frac{\tau}{\sqrt{d}} Q_1 \bar{A}_{\omega i} \\ \tau Q_2 \bar{A}_{di} & \tau Q_2 \bar{A}_{\omega i} \\ L_{di} & -L_{\omega i} \end{bmatrix},$$

$$\Psi_{33} = \mathrm{diag}\{-Q_1, -Q_2, -I\},$$

$$\Gamma = P_i \bar{A}_i + \bar{A}_i^T P_i + \sum_{j=1}^{N} \pi_{ij} P_j - d * Q_1 - Q_2,$$

$$R_i = \begin{bmatrix} R_{i11} & * & * & * \\ R_{i21} & R_{i22} & * & * \\ \vdots & \vdots & \ddots & \vdots \\ R_{id1} & R_{id2} & \cdots & R_{idd} \end{bmatrix}.$$

Proof Define the new vector

$$\zeta^T(t) = \left[e^T(t) \quad e^T(t - \tfrac{\tau}{d}) \quad e^T(t - \tfrac{2\tau}{d}) \quad \cdots \quad e^T(t - \tfrac{(d-1)\tau}{d}) \right]$$

and choose the Lyapunov functional as

$$V(x_t, \theta_t) = \sum_{i=1}^{4} V_i(x_t, \theta_t), \tag{49}$$

where

$$V_1(x_t, \theta_t) = e^T(t) P(\theta_t) e(t),$$

$$V_2(x_t, \theta_t) = \int_{t-\frac{\tau}{d}}^{t} \zeta^T(s) R(\theta_t) \zeta(s)\, ds,$$

$$V_3(x_t, \theta_t) = \tau \int_{-\frac{\tau}{d}}^{0} \int_{t+s}^{t} \dot{e}^T(v) Q_1 \dot{e}(v)\, dv\, ds + \tau \int_{-\tau}^{0} \int_{t+s}^{t} \dot{e}^T(v) Q_2 \dot{e}(v)\, dv\, ds,$$

$$V_4(x_t, \theta_t) = \int_{-\frac{\tau}{d}}^{0} \int_{t+s}^{t} \dot{\zeta}^T(v) Z \dot{\zeta}(v)\, dv\, ds.$$

Then, (47) can be obtained similarly to the proof of Theorem 1. □

4 H_∞ Filter Design

In the following, we are seeking to design the H_∞ filtering based on Theorems 1 and 2.

Theorem 3 *For some given constants $0 \le \tau_m \le \tau_M$ and γ, system (6) is ESMSS with a prescribed H_∞ performance γ if there exist $P_i > 0$, $Q_1 > 0$, $Q_2 > 0$, $Q_3 > 0$, $Q_{4i} > 0$, $R_1 > 0$, $R_2 > 0$, $R_3 > 0$, $Z_1 > 0$, $Z_2 > 0$, $Z_3 > 0$, M_{ik}, N_{ik}, T_{ik}, S_{ik}, and $\bar{G}_i$ $(i \in \mathcal{S}, k = 1, 2, \dots, 6)$ with appropriate dimensions such that the following LMIs hold for a given $\varepsilon > 0$:*

$$\hat{\Psi} = \begin{bmatrix} \hat{\Psi}_{11} + \Gamma + \Gamma^T & * & * \\ \hat{\Psi}_{21} & \hat{\Psi}_{22} & * \\ \Psi_{31}(s) & 0 & -R_2 \end{bmatrix} < 0, \quad s = 1, 2, \tag{50}$$

$$\hat{\Omega} = \begin{bmatrix} \hat{\Omega}_{11} + \Upsilon + \Upsilon^T & * & * \\ \hat{\Psi}_{21} & \hat{\Psi}_{22} & * \\ \Omega_{31}(s) & 0 & -R_3 \end{bmatrix} < 0, \quad s = 1, 2, \tag{51}$$

$$\sum_{j=1}^{N} \pi_{ij} Q_{4j} \le Z_k, \quad k = 1, 2, 3, \tag{52}$$

where

$$\hat{\Psi}_{11} = \begin{bmatrix} \hat{\Pi}_1 & * & * & * & * & * \\ A_{di}^T P_i + C_{di}^T \bar{G}_i^T & -(1-\mu) Q_{4i} & * & * & * & * \\ R_1 & 0 & -Q_1 - R_1 & * & * & * \\ 0 & 0 & 0 & -Q_2 - \frac{R3}{\delta} & * & * \\ 0 & 0 & 0 & \frac{R3}{\delta} & -Q_3 - \frac{R3}{\delta} & * \\ -A_{\omega i}^T P_i - C_{\omega i}^T \bar{G}_i^T & 0 & 0 & 0 & 0 & -\gamma^2 I \end{bmatrix},$$

$$\hat{\Omega}_{11} = \begin{bmatrix} \hat{\Pi}_1 & * & * & * & * & * \\ A_{di}^T P_i + C_{di}^T \bar{G}_i^T & -(1-\mu) Q_{4i} & * & * & * & * \\ R_1 & 0 & -Q_1 - R_1 - \frac{R2}{\delta} & * & * & * \\ 0 & 0 & \frac{R2}{\delta} & -Q_2 - \frac{R2}{\delta} & * & * \\ 0 & 0 & 0 & 0 & -Q_3 & * \\ -A_{\omega i}^T P_i - C_{\omega i}^T \bar{G}_i^T & 0 & 0 & 0 & 0 & -\gamma^2 I \end{bmatrix},$$

$$\hat{\Psi}_{21} = \begin{bmatrix} \tau_m P_i A_i + \tau_m \bar{G}_i C_i & \tau_m P_i A_{di} + \tau_m \bar{G}_i C_{di} & 0 & 0 & 0 & -\tau_m P_i A_{\omega i} - \tau_m \bar{G}_i C_{\omega i} \\ \sqrt{\delta} P_i A_i + \sqrt{\delta} \bar{G}_i C_i & \sqrt{\delta} P_i A_{di} + \sqrt{\delta} \bar{G}_i C_{di} & 0 & 0 & 0 & -\sqrt{\delta} P_i A_{\omega i} - \sqrt{\delta} \bar{G}_i C_{\omega i} \\ \sqrt{\delta} P_i A_i + \sqrt{\delta} \bar{G}_i C_i & \sqrt{\delta} P_i A_{di} + \sqrt{\delta} \bar{G}_i C_{di} & 0 & 0 & 0 & -\sqrt{\delta} P_i A_{\omega i} - \sqrt{\delta} \bar{G}_i C_{\omega i} \\ L_i & L_{di} & 0 & 0 & 0 & -L_{\omega i} \end{bmatrix},$$

$$\hat{\Psi}_{22} = \operatorname{diag}\{-2\varepsilon P_i + \varepsilon^2 R_1, -2\varepsilon P_i + \varepsilon^2 R_2, -2\varepsilon P_i + \varepsilon^2 R_3, -I\},$$

$$\hat{\Pi}_1 = P_i A_i + A_i^T P_i + \bar{G}_i C_i + C_i^T \bar{G}_i^T + Q_1 + Q_2 + Q_3 + Q_{4i}$$
$$- R_1 + \tau_m Z_1 + \delta Z_2 + \delta Z_3 + \sum_{j=1}^{N} \pi_{ij} P_j,$$

and Γ, Υ, $\Psi_{31}(s)$, and $\Omega_{31}(s)$ $(s = 1, 2)$ are defined as Theorem 1.

Moreover, the filter gain in the form of (5) is given as follows:

$$G_i = P_i^{-1} \bar{G}_i. \tag{53}$$

Proof Defining $\bar{G}_i = P_i G_i$, from (15), (16) and using Schur complement, the matrix inequalities (15) and (16) hold if and only if

$$\hat{\Psi} = \begin{bmatrix} \hat{\Psi}_{11} + \Gamma + \Gamma^T & * & * \\ \hat{\Psi}_{21} & \breve{\Psi}_{22} & * \\ \Psi_{31}(s) & 0 & -R_2 \end{bmatrix} < 0, \quad s = 1, 2, \tag{54}$$

$$\hat{\Omega} = \begin{bmatrix} \hat{\Omega}_{11} + \Upsilon + \Upsilon^T & * & * \\ \hat{\Psi}_{21} & \breve{\Psi}_{22} & * \\ \Omega_{31}(s) & 0 & -R_3 \end{bmatrix} < 0, \quad s = 1, 2, \tag{55}$$

where

$$\breve{\Psi}_{22} = \operatorname{diag}\{-P_i R_1^{-1} P_i, -P_i R_2^{-1} P_i, -P_i R_3^{-1} P_i, -I\}.$$

Since

$$\left(R_k - \varepsilon^{-1} P_i\right)^{-1}\left(R_k - \varepsilon^{-1} P_i\right) \geq 0, \quad k = 1, 2, 3 \tag{56}$$

we get

$$-P_i R_k^{-1} P_i \leq -2\varepsilon P_i + \varepsilon^2 R_k, \quad k = 1, 2, 3. \tag{57}$$

Substituting $-P_i R_k^{-1} P_i$ with $\varepsilon P_i + \varepsilon^2 R_k$ into (54) and (55), we obtain (50) and (51); hence, if (50) and (51) hold, then (15) and (16) hold, and from above proof we have $G_i = P_i^{-1} \bar{G}_i$. This completes the proof. □

Remark 4 Inequality (57) is used to bound the term $-P_i R_k^{-1} P_i$ in (54) and (55). This step can be improved by adopting the cone complementary algorithm [8], which is popular in recent control designs. Here the scaling parameter $\varepsilon > 0$ can be used to improve the conservatism in Theorem 3.

Similarly, the following result can be obtained for system (46).

Theorem 4 *For some given constants τ, d, and γ, system (46) is ESMSS with a prescribed H_∞ performance γ if there exist $P_i > 0$, $Q_1 > 0$, $Q_2 > 0$, $R_i > 0$, $Z > 0$, and $\bar{G}_i$ $(i \in \mathcal{S})$ with appropriate dimensions such that the following LMIs hold for a given $\varepsilon > 0$:*

$$\Psi = \begin{bmatrix} \hat{\Psi}_{11} + R_i + \frac{\tau}{d} Z & * & * \\ \hat{\Psi}_{21} & \Psi_{22} & * \\ \hat{\Psi}_{31} & \hat{\Psi}_{32} & \hat{\Psi}_{33} \end{bmatrix} < 0, \tag{58}$$

$$\sum_{j=1}^{N} \pi_{ij} R_j \leq Z, \tag{59}$$

where

$$\hat{\Psi}_{11} = \begin{bmatrix} \hat{\Gamma} & * & * & * & * \\ d*Q_1 & -R_{i11} - d*Q_1 & * & * & * \\ 0 & -R_{i21} & -R_{i22} & * & * \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & -R_{i(d-1)1} & -R_{i(d-1)2} & \cdots & -R_{i(d-1)(d-1)} \end{bmatrix},$$

$$\hat{\Psi}_{21} = \begin{bmatrix} A_{di}^T P_i + C_{di}^T \bar{G}_i^T + Q_2 & -R_{id1} & -R_{id2} & \cdots & -R_{id(d-1)} \\ -A_{\omega i}^T P_i - C_{\omega i}^T \bar{G}_i^T & 0 & 0 & \cdots & 0 \end{bmatrix},$$

$$\hat{\Psi}_{31} = \begin{bmatrix} \frac{\tau}{\sqrt{d}} P_i A_i + \frac{\tau}{\sqrt{d}} \bar{G}_i C_i & 0 & 0 & \cdots & 0 \\ \tau P_i A_i + \tau \bar{G}_i C_i & 0 & 0 & \cdots & 0 \\ L_i & 0 & 0 & \cdots & 0 \end{bmatrix},$$

$$\hat{\Psi}_{32} = \begin{bmatrix} \frac{\tau}{\sqrt{d}} P_i A_{di} + \frac{\tau}{\sqrt{d}} \bar{G}_i C_{di} & -\frac{\tau}{\sqrt{d}} P_i A_{\omega i} - \frac{\tau}{\sqrt{d}} \bar{G}_i C_{\omega i} \\ \tau P_i A_{di} + \tau \bar{G}_i C_{di} & -\tau P_i A_{\omega i} - \tau \bar{G}_i C_{\omega i} \\ L_{di} & -L_{\omega i} \end{bmatrix},$$

$$\hat{\Psi}_{33} = \mathrm{diag}\{-2\varepsilon P_i + \varepsilon^2 Q_1, -2\varepsilon P_i + \varepsilon^2 Q_2, -I\},$$

$$\hat{\Gamma} = P_i A_i + A_i^T P_i + \bar{G}_i C_i + C_i^T \bar{G}_i^T + \sum_{j=1}^{N} \pi_{ij} P_j - d*Q_1 - Q_2,$$

and Ψ_{22} and R_i are defined as in Theorem 2.

Moreover, the filter gain in the form of (5) is given as follows:

$$G_i = P_i^{-1} \bar{G}_i. \tag{60}$$

5 Example

In this section, a well-studied example is used to illustrate the effectiveness of the approaches proposed in this paper.

Example 1 Consider a Markovian jump system in (1) with two modes and the following parameters [30]:

$$\bar{A}_1 = \begin{bmatrix} -2.2460 & -1.4410 \\ -1.5937 & -2.9289 \end{bmatrix}, \qquad \bar{A}_2 = \begin{bmatrix} -1.8999 & 0.8156 \\ 0.6900 & -0.7881 \end{bmatrix},$$

$$\bar{A}_{d1} = \begin{bmatrix} -0.7098 & 1.1908 \\ 0.6686 & -3.2025 \end{bmatrix}, \qquad \bar{A}_{d2} = \begin{bmatrix} -1.5198 & -1.6041 \\ -0.1567 & -1.2427 \end{bmatrix},$$

$$\bar{A}_{\omega 1} = \begin{bmatrix} 0.0403 \\ 0.6771 \end{bmatrix}, \qquad \bar{A}_{\omega 2} = \begin{bmatrix} 0.5689 \\ -0.2556 \end{bmatrix},$$

$$L_1 = \begin{bmatrix} -0.3775 & -0.2959 \end{bmatrix}, \qquad L_2 = \begin{bmatrix} -1.4751 & -0.2340 \end{bmatrix},$$

$$L_{d1} = \begin{bmatrix} 0 & 0 \end{bmatrix}, \qquad L_{d2} = \begin{bmatrix} 0 & 0 \end{bmatrix}, \qquad L_{\omega 1} = -0.1184, \qquad L_{\omega 2} = -0.3148.$$

Table 1 Maximum allowable values of τ_M for $\tau_m = 0$, $\mu = 0.5$, and $\pi_{22} = -0.6$

γ	0.4	0.8	1.2	1.6
τ_M by [30]	0.3359	0.3975	0.4116	0.4181
τ_M by Theorem 1	0.3666	0.4293	0.4450	0.4524

Table 2 Maximum allowable values of τ_M for $\mu = 1$ and $\pi_{22} = -1$

γ	0.4	0.8	1.2	1.6
τ_M by [30]	0.2690	0.2833	0.2852	0.2858
τ_M by Theorem 1	0.3616	0.4206	0.4353	0.4422

Suppose that the transition probability matrix is given by $\pi_{11} = -3$.

For several values of μ and π_{22}, the computation results of τ_M are listed in Tables 1–2. Obviously, for the same conditions of the time delay, our results are less conservative than those in the existing references.

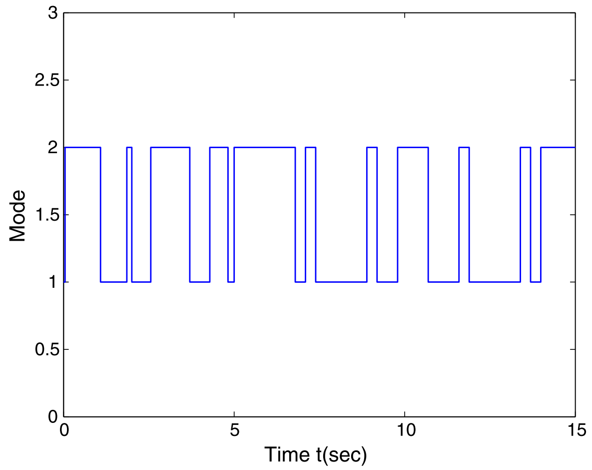
To illustrate the proposed method on filtering design, two examples are considered as follows.

Example 2 Consider linear Markovian jump systems in the form (1) with two modes. For modes 1 and 2, the dynamics of systems are described as

$$
\begin{aligned}
A_1 &= \begin{bmatrix} -3 & 1 & 0 \\ 0.3 & -2.5 & 1 \\ -0.1 & 0.3 & -3.8 \end{bmatrix}, \qquad A_{d1} = \begin{bmatrix} -0.2 & 0.1 & 0.6 \\ 0.5 & -1 & -0.8 \\ 0 & 1 & -2.5 \end{bmatrix}, \\
A_{\omega 1} &= \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \\
C_1 &= \begin{bmatrix} 0.8 & 0.3 & 0 \end{bmatrix}, \qquad C_{d1} = \begin{bmatrix} 0.2 & -0.3 & -0.6 \end{bmatrix}, \qquad C_{\omega 1} = 0.2, \\
L_1 &= \begin{bmatrix} 0.5 & -0.1 & 1 \end{bmatrix}, \qquad L_{d1} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}, \qquad L_{\omega 1} = 0, \\
A_2 &= \begin{bmatrix} -2.5 & 0.5 & -0.1 \\ 0.1 & -3.5 & 0.3 \\ -0.1 & 1 & -2 \end{bmatrix}, \qquad A_{d2} = \begin{bmatrix} 0 & -0.3 & 0.6 \\ 0.1 & 0.5 & 0 \\ -0.6 & 1 & -0.8 \end{bmatrix}, \\
A_{\omega 2} &= \begin{bmatrix} -0.6 \\ 0.5 \\ 0 \end{bmatrix}, \\
C_2 &= \begin{bmatrix} 0.5 & 0.2 & 0.3 \end{bmatrix}, \qquad C_{d2} = \begin{bmatrix} 0 & -0.6 & 0.2 \end{bmatrix}, \qquad C_{\omega 2} = 0.5, \\
L_2 &= \begin{bmatrix} 0 & 1 & 0.6 \end{bmatrix}, \qquad L_{d2} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}, \qquad L_{\omega 2} = 0.
\end{aligned}
$$

Suppose that the transition probability matrix is given by $\pi = \left[\begin{smallmatrix} -0.5 & 0.5 \\ 0.3 & -0.3 \end{smallmatrix}\right]$ and the initial conditions $x(0) = [-0.2\ 0.3\ 0.9]^T$, $\hat{x}(0) = [-0.4\ 0.6\ 1.8]^T$.

Fig. 1 Operation modes



By using Theorem 3 we can get the maximum time delay $\tau_M = 5.0944$ for $\tau_m = 0.1$, $\mu = 0.4$, $\varepsilon = 10$, and $\gamma = 0.5$. The corresponding filter parameters are given as

$$G_1 = \begin{bmatrix} 0.3837 \\ -1.4310 \\ -3.1457 \end{bmatrix}, \qquad G_2 = \begin{bmatrix} 2.2251 \\ -0.1772 \\ -0.6581 \end{bmatrix}.$$

To illustrate the performance of the designed filter, choose the disturbance function as follows:

$$\omega(t) = \begin{cases} 0.1, & 2 < t < 5, \\ 0, & \text{otherwise}. \end{cases}$$

With this filter, Figs. 1–4 show the operation modes of the MJS, interval time-varying delay, estimated signal $z(t)$, $\tilde{z}(t)$, and estimated signals error $\eta(t) = z(t) - \tilde{z}(t)$, respectively.

Example 3 Consider linear Markovian jump systems in the form (46) with two modes. For modes 1 and 2, the parameters of system (46) are described as in Example 2.

This system is nominal one considered in [9]. By Theorem 4, when $\varepsilon = 10$ and $\gamma = 1.2$, for different d, the computation results of τ are listed in Table 3. Obviously, for the same conditions for the time delay, our method can lead to less conservative results.

When $d = 4$, $\varepsilon = 10$, and $\gamma = 1.2$, we get the maximum time delay $\tau = 2.2179$, and the corresponding filter parameters are given by

$$G_1 = \begin{bmatrix} 0.5492 \\ 0.2716 \\ -4.2943 \end{bmatrix}, \qquad G_2 = \begin{bmatrix} 1.3004 \\ -1.0214 \\ -1.0646 \end{bmatrix}.$$

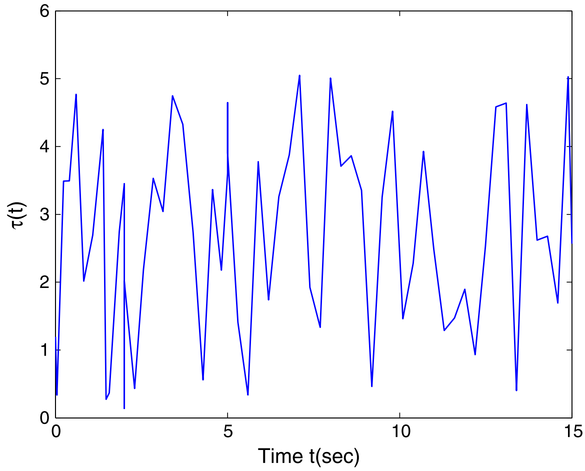


Fig. 2 Interval time-varying delay

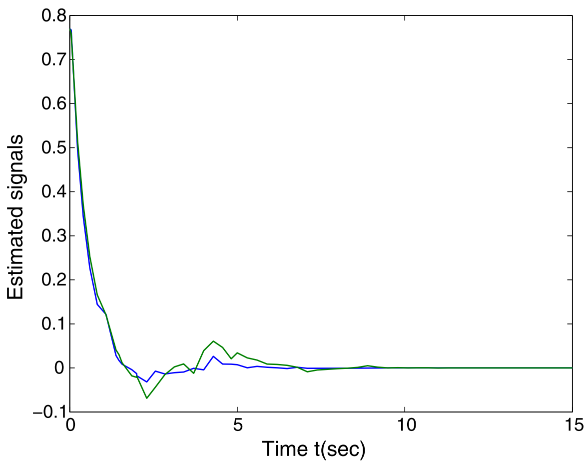


Fig. 3 Estimated signals $z(t)$ and $\tilde{z}(t)$

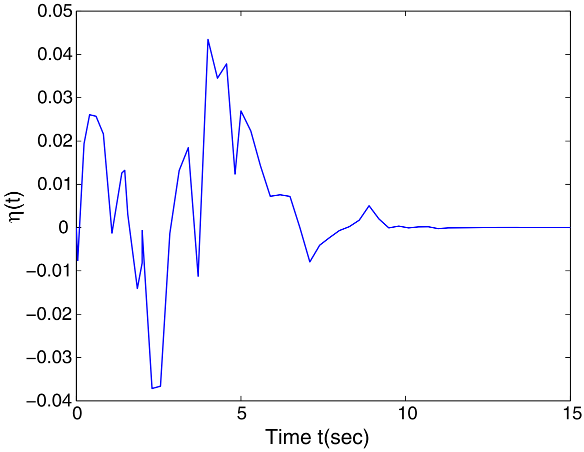


Fig. 4 Estimated signals error $\eta(t) = z(t) - \tilde{z}(t)$

Table 3 Maximum allowable values of τ for $\varepsilon = 10$ and $\gamma = 1.2$

	[9]	$d = 2$	$d = 3$	$d = 4$	$d = 5$
τ	1.9195	2.0784	2.1886	2.2179	2.2220

6 Conclusion

In this paper, we have studied a class of H_∞ filter design for Markovian jump systems with time-varying delays via manipulating a new Lyapunov function and using the convexity property of the matrix inequality. By using the piecewise analysis method, LMI-based sufficient conditions for the existence of the desired H_∞ filter have been derived, which can lead to much less conservative analysis results. Finally, numerical examples have been carried out to demonstrate the effectiveness of the proposed method.

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